Cosmological Model with A Nonhomogeneous Cosmic Time

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The principle of consistent relativity for establishing of the dynamics equation is firstly introduced in [1] and then we can discuss the dynamics law of the universe by use of the method of inertial reference frame. We strictly refer to the establishing of the geometrized equation of the gravity on the solar system scale, and gravitational physics effect opened up form the Schwarzschild space-time. Then the metric of the gravity-geometrized space-time is restated. We introduce two geometrical variables $\{b(t), a(t)\}$ into the cosmological metric according to the varying gravity in the universe. For $b(t)$ is the gravitational time dilation factor, the running of $b(t)$ symbolizes the nonhomogeneous evolution of the cosmic time $(b(t)dt)$ within the same time interval (dt) for the present observer rest on the Earth. On this base, we investigate the corresponding fundamental equations in our cosmological model with the nonhomogeneous cosmic time, and lastly, the *chain condition* between the geodesics of an instantaneous metric solution and the distribution of the gravitational matter in the next moment is also carried out.
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INTRODUCTION

In Ref.[1], the authors have pointed out that the most essential conception for the "inertial reference frame" is that the dynamics relativity must be maintained from the aspect of symmetry and causality. Once it has been done in all inertial reference frames, we would have the same form of the dynamics law in these reference frames. Furthermore, we can make a adjustment of the causalities on the both sides of the dynamics equation in any inertial reference frame among of them, then first law of motion would be automatically carried out in all the inertial reference frames.

In the spirit of above proposition, we have discussed in detail the nature way to realize the dynamics relativity from the aspect of symmetry and causality, and proposed the principle of consistent relativity for establishing the dynamics equation. The start point of this proposition is based on such a fact that our observation of gravity executed on all objects in celestial system is local, which also is the origin of relativity. In fact, the influences of the gravity from out-space of the system on the inner relative motion can be almost erased by the global motion of the system. Thus, in any gravity-free system, the count of gravitation interaction should be treated as a relative physical quantity, to all the observers in this system.

It is well known that the kinetic quantity appearing in dynamic equation must be a relative physical quantity. Therefore, to keep the invariant form of the dynamic equation, we need to start from the idea of symmetry on the relativity in both sides of dynamic equation. From the viewpoint of symmetry, we know that the speed of the observed particle is relative, and the accelerated speed of the observed particle also is relative; thus, to keep the invariance of the dynamic equation, the force to be counted also is relative. While in fact, there exists such a celestial system with natural superiority, which means the relativity in different level of gravity-bounded system can be counteracted globally and symmetrical with high precision, the essential reason owe to the equivalence of gravitational mass and inertial mass.

In Ref.[1], the author realized the definition of a kind of natural inertial reference frame through the introduction of the gravity relativity, and proposed that the minimal gravity-free observer can construct the inertial reference frame naturally. The relativity symmetry can be realized, only for the dynamic equation established on this kind of observer. Because we can find physical inertial reference frame for all systems to be observed, but the system of gravity to be counted must be consistent with the selection of its inertial reference frame, thus we can call it as the principle of consistent relativity in dynamics.

On this base, we are going to demonstrate that the geometric equivalence of gravity in solar system meets the principle of consistent relativity, which means that the count of relative gravity in solar system is consistent with the choosing of mass center of solar system as the origin. In this regard, the cosmological metric should be established on the inertial reference frame of universal background. The main purpose of this paper is to discuss the evolution of universe and its observation theory by use of the method of inertial reference frame with the gravity physics disclosed in the Schwarizild case.

GRAVITATIONAL PHYSICS IN THE SCHWARZSCHILD CASE

As it is well known, the gravity testing of Einstein's field equation have achieved great successes on the solar scale. Therefore, the establishment of the cosmological metric, besides resorting to the Cosmological Principle, must be also resorting to the physics in the Schwarzschild metric on the solar scale. Above all, the scope of the interaction under the consideration in these gravity testings is the gravitational interaction in the inner region of the sun system, hence the mass center of the sum system should be chosen as the origin of the inertial reference, and actually we have so done it before. That is to say, we have a background inertial reference on the solar scale,

$$ds^2 = -dt'^2 + d\vec{x'}^2. \tag{1}$$

Firstly, it is introduced in mathematics because all the space-time intervals in the inner region of the sun system should be measured by the observer rest on the Earth with their proper scales. Secondly, the metric is mathematically homogeneous and isotropic which has just corresponded to a physical assumption: the proper scale of the space-time on different space points would be equal to each other if it is considered only in the background space-time where doesn't exist any gravitational interaction. Then we can geometrize the gravity in the inner region of the sum system on the basis of this inertial metric (1) and we directly write down its correct form—the Schwarzschild metric,

$$ds^2 = -B^2(r')dt'^2 + A^2(r')dr'^2 + r'^2 d\theta'^2 + r'^2 sin^2\theta' d\varphi'^2. \tag{2}$$

It is the spherical reference frame with the origin at the mass center of the sum system, which can be seen from the theoretical analysis of solar gravity experiments. In addition, t', r' are still coordinates of the background reference frame (1). Therefore, after geometrized, the gravity effect is completely demonstrated by the difference of the proper scale of the space-time on different space points under the solar gravity. The strength of gravity is equivalently geometrized into the actual proper space-time scale measured by the observer test on the Earth at the same moment. In this regard, as well as we have been identifying dt' as the reading of proper clock on the observer rest on the Earth, we can identify $B(r')dt'$ as the reading of the proper clock on different space points (Both are read out at the same moment). As we know, $B(r') < 1$ in the Schwarzschild case, which has just meant that the running of the proper time is absolutely decelerated under the action of the gravity. Furthermore, $B(r')$ is decreasing with the decrease of r' where the gravity becomes more and more strong. To the observer rest on the Earth, he will have $B(r'_1)dt' < B(r'_0)dt'$ at the same moment if $r'_1 < r'_0$. That is to say, the stronger the gravity is, the longer the period of proper events behaves (keep mind that is relative to the same observer). Then the gravitational redshift in the Schwarzschild case is theoretically carried out.

The above discussion that the gravity equals the change of the space-time interval of the unit proper events is one explanation of the gravitational redshift in the Schwarzschild case. The essential reason of the redshift of a photon is the difference of the total energy gained from the same kind of energy transition under different gravity field. the motion would not result in the redshift, once the photon is generated. That is to say, in the first scenario of the gravity geometry theory, the discrepancy of the gravitational space-time scale is a true phenomenon in physics, which not only take effect on the motion of the observed particle, but also take effect for all the proper events.

The another explanation of the gravity geometry theory still not capable being ruled out is that the motion of any photon in gravity field results in the redshift. In this scenario, what can be showed in the Schwarzschild metric is nothing but the kinematical equivalence of the gravity on the background of vacuum space-time in the investigating of the motion of a gravity-free particle. It is an equivalent description in mathematics and only take effect on the motion of the particle. But in fact the true space-time with the gravity field is still homogeneous and isotropic, which means that the space-time scale correspond to local proper events at different space points are uniform. However, there exists one point different with Newtonian mechanism and Special relativity that the motion of any particle in the gravity field is also obeying the geometrical variation principle. No matter of what explanation is, they would be equivalent with each other on the investigation of the motion of particles, which is just the physics concerned in this paper. Considering that the principle of space-time interval invariant is consistent with the proper energy conservation law in Special relativity, we prefer to investigate the gravitational redshift of the photon in the Schwarzschild case by use of a simulated approach of the proper energy conservation law in gravity field. First, the kinematics theory is pure of geometry and the essence of the motion would not be changed by performing a general coordinates transformation. Therefore, it is always available to redefine the scale of Schwarzschild space-time obtaining a mathematical flat space-time,

$$ds^2 = -dT^2 + dR^2 + R^2 d\theta^2 + R^2 sin^2\theta d\varphi^2. \tag{3}$$

In this mathematical flat space-time, we may regard its coordinates are equivalently measured by the physical events occurred on different space points (first kind of explanation), so kinematical quantities described by this set of coordinates can be named as the event reading. The radial velocity of the particle satisfies

$$V = \frac{dR}{dT} = \frac{a(r')dr'}{b(r')dt} = \frac{a(r')}{b(r')}v. \tag{4}$$

As long as the motion of the gravity-free particle is always to obey the same principle for different observers, the traditional mass transformation caused by the speed can still be quoted here,

$$m = \frac{m_0}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{m_0}{\sqrt{1-\frac{a^2(r')v^2}{b^2(r')c^2}}}. \tag{5}$$

We assume that the light source is moving apart from the observer in the speed of v and the frequency of a photon, when emitted, is ν_0, the frequency when received by the earth observer is ν. We may take such a equivalence for the photon,

$$h\nu = mc^2 \tag{6}$$
$$h\nu_0 = m_0c^2 \tag{7}$$
$$p = mv. \tag{8}$$

here m_0 is the kinematical mass of the photon from the viewpoint of the light source. In general, as long as the rest mass of the particle equals with each other in different background of gravity, the proper energy conservation law is in the similar form of Special relativity,

$$-m_0^2c^4 = -m_1^2c^4 + p_1^2c^2 = -m_2^2c^4 + p_2^2c^2. \tag{9}$$

In the spirit of the geometrical equivalence for the gravity, the kinematical description of the gravitational potential energy may be written down as

$$\frac{1}{2}mv^2 = \frac{1}{2m}p^2 \Leftrightarrow \frac{GMm}{r'}. \tag{10}$$

Substituting (10) into (9), we may propose the following proper energy conservation law for gravity-free particle,

$$-m_0^2c^4 + \frac{2GMm_0^2}{r'_0}c^2 = -m^2c^4 + \frac{2GMm^2}{r'}c^2. \tag{11}$$

Therefore,

$$h^2\nu^2 + h^2\nu_0^2\frac{2GM}{c^2r'_0} = h^2\nu_0^2 + h^2\nu^2\frac{2GM}{c^2r'}. \tag{12}$$

Then we have

$$\frac{\nu_0^2}{\nu^2} = \frac{1-\frac{2GM}{c^2r'}}{1-\frac{2GM}{c^2r'_0}} = \frac{b^2(r')}{b^2(r'_0)}, \tag{13}$$
$$\frac{\nu_0}{\nu} = \sqrt{\frac{1-\frac{2GM}{c^2r'}}{1-\frac{2GM}{c^2r'_0}}}. \tag{14}$$

After that, the exact expression of the redshift in the Schwarzschild is recovered. The kinematical equivalence of the gravitational potential energy takes the following form,

$$p^2c^2|_{(\mathrm{r'})} = \frac{2GMm^2}{r'}c^2 = (1-b^2(r'))m^2c^4. \tag{15}$$

In addition, it should be noted that the physics demonstrated by the change of the coordinates scale on the inertial reference frame (2) is only the kinematical equivalence of the gravity. Therefore, to practical transformation of the space-time interval in gravity field, we must take further consideration of the effect Lorenz transformation, besides if gravity takes effect. In a similar way, the counting of the effect of Doppler is also required in the calculating of practical redshift of a photon.

COSMOLOGICAL MODEL WITH A NONHOMOGENEOUS COSMIC TIME

In cosmology, the object we deal with is all the ordinary matter and the all gravitational interaction in the universe. In the spirit of the symmetry of the relativity, the background space-time on which we could geometrize all the gravity in our practical universe must be the space-time of an entirely empty universe. Obviously, the background space-time to the cosmological gravity is just the Minkowski space (conversely, which may be regarded as the definition of the Minkowski space). To the present observer rest on the Earth, the Minkowski metric as a background can also be available just as an instantaneous observer in the entirely empty universe. Because the observer rest on the Earth still is in gravity free at the large scale. Consequently, performed by an instantaneous rescaling, the metric of the background space-time with the scale of the present observer rest on the Earth can be written as

$$ds^2 = -dt^2 + d\vec{x}^2. \tag{16}$$

The next step is to constitute a geometrized space-time model according to the Cosmological Principle. Therefore, we can establish a connection between the comoving coordinates of the geometrized metric and the background reference frame of the present Earth observer to describe the evolution of the gravitational matter in the background space-time. On the other hand, as it is well known, a stationary gravity field must result in the discrepancy between the clocks rest on different space points. Along this way, it is reasonable to extrapolate that the running of the clock rest on the same space point will also be nonsynchronous (with the time) due to the varying gravity executed on it. Therefore, the geometrized metric of the cosmological gravity can be given by

$$ds^2 = -b^2(t)dt^2 + a^2(t)[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2sin^2\theta d\varphi^2]. \tag{17}$$

Where $\{t, r, \theta, \varphi\}$ still are the background coordinates of the inertial observer of universe, and they no longer denote commoving coordinate, but the distance variance between the commoving points can be fully represented by the dynamic law. In addition, $a(t)$ has just characterized the homogeneousness and isotropy of the space and the proper time scale $b(t)$ bears the effect of gravitational time dilation. For t is the background time of the inertial reference frame (16), the running of $b(t)$ symbolizes the nonhomogeneous evolution of the cosmic time $b(t)dt$ within the same time interval dt for the present observer rest on the Earth. In addition, it can be checked and approved that the world line of the commoving coordinates point ($(r, \theta, \varphi) = const.$) is just the one kind of the geodesics [2] in the space-time with metric (17). What's more, according to solution of Schwarzschild metric, the solution of cosmological metric should be executed under the expression of inertial observer in universe. According to the line element (17), the non-zero components of its Ricci tensor are given by

$$R_{00} = -3\frac{\ddot{a}}{a} + 3\frac{\dot{a}\dot{b}}{ab}; \tag{18}$$

$$R_{ij} = \frac{1}{b^2}(2\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a} - \frac{\dot{a}\dot{b}}{ab})g_{ij} + \frac{2k}{a^2}g_{ij}. \tag{19}$$

To be consistent with the symmetries implicated in the metric (17), the total stress-energy tensor should be diagonal and may be simply realized by that of a perfect fluid $< T^{\mu}_{\nu} >_{grav} = diag\{\rho, -p, -p, -p\}$. Then, the fundamental equations of the cosmology may be taken as

$$\frac{\dot{a}^2}{a^2b^2} + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho + \Lambda); \tag{20}$$

$$\frac{1}{b^2}(\frac{\ddot{a}}{a} - \frac{\dot{a}\dot{b}}{ab}) = -\frac{4\pi G}{3}(\rho + 3p) + \frac{8\pi G}{3}\Lambda. \tag{21}$$

It is well known, in Einstein's field equation, there are ten independent components in the whole tensor equation, which is just corresponding to the 10 freedom numbers in the symmetric metric. Just as the ${g^{\mu\nu}}_{;\mu} \equiv 0$ is nothing but a definition on the affine connection and do not change the number of the freedom of the symmetric metric, the conversation law ${T^{\mu\nu}}_{;\mu} = 0$ is not capable influencing the completeness of the gravitational field equation. The completeness of the above equations can be carried out due to a fact that the introduction of the inertial reference frame in our theory has been actually in equivalence to the introduction of the coordinates conditions. Therefore, the 2 geometric variables which we have introduced from previous physical considerations just correspond to the 2 freedom numbers of the perfect fluid model of the ordinary matter in a homogeneous isotropic universe, and the equivalence between the gravity and the geometrization language is naturally carried out in such a metric form (17).

According to the assumption of cosmological homogeneous isotropy and the Poincare relativity of universal background observer, we can representatively choose an adjacent commoving system that consists of the earth and a certain light source, where the propagation law is applicable to all local regions in the universe. To simplify the mathematic representation, we assume that commoving points move along radial direction. Because currently existed universe is in expanding, the initial speed of commoving points must be not zero. Then we can denote the initial speed of the commoving point including the earth at the moment t_e of decoupling from other interaction to be v_e; The initial speed of the light source relative to inertial observer of universe is v_s , when it start to be in gravity free at t_s. Similar to the derivation of the mathematical flat time-space from the redefinition of the coordinates scale in Schwarzschild case, the redefined speed of the cosmological space-time characterized by Eq.(17), which satisfies the Lorenz transformation, can be expressed as follows:

$$V_e(t) = \frac{a(t)dr}{b(t)dt} = \frac{a(t)}{b(t)}v_e(t); \tag{22}$$

$$V_s(t) = \frac{a(t)dr}{b(t)dt} = \frac{a(t)}{b(t)}v_s(t) \tag{23}$$

From the aspect of theory, the motion of the gravity-free particle should be obtained according to the geometrical variation principle, no matter the first explanation or the second explanation of the gravity geometry theory. But, there exist difficulty for directly applying this principle as for the non-stationary cosmological metric. Thus, we hope to realize the geometrical discussion of the dynamics law of cosmological commoving points, in resorting to the geometrical language of invariant space-time interval adopted by special relativity. At random time t, no matter the commoving particle with mass is accelerated in any way, according to the proper energy conservation of gravity-free particle in gravitational geometry, we have the following relation

$$-m^2|_{(\mathrm{IS})}c^4 + p^2c^2|_{(\mathrm{IKEGF})} = -m^2c^4 + p^2c^2|_{(\mathrm{KEGF})}. \tag{24}$$

here IS denotes "initial speed" and $IKEGF$ denotes "the kinematical equivalence for the gravity field around the particle at the initial moment ". And $KEGF$ denotes "the kinematical equivalence for the gravity field around the particle at the moment t". In further study, after referring to the connection between the change of the gravity potential energy in the Schwarzschild space-time and the gravitational red-shift of photon, we obtain the following relation for the gravity-free point of the earth.

$$p^2c^2|_{(\mathrm{KEGF})} = (1-b^2(t))\frac{m_0^2}{1-\frac{V_e^2(t)}{c^2}}c^4. \tag{25}$$

$$p^2c^2|_{(\mathrm{IKEGF})} = (1-b^2(t_e))\frac{m_0^2}{1-\frac{V_e^2}{c^2}}c^4. \tag{26}$$

$$m^2c^4|_{(\mathrm{IS})} = \frac{m_0^2}{1-\frac{V_e^2}{c^2}}\cdot c^4 = \frac{m_0^2}{1-\frac{a(t)v_e^2}{b(t)c^2}}\cdot c^4. \tag{27}$$

From the kinematics theory of the gravity-free point, we obtain the speeds of earth and light source $v_e(t)$ and $v_s(t)$ at the moment t,

$$\frac{a(t)}{b(t)}v_e(t) = V_e(t) = \sqrt{c^2 - \frac{b^2(t)}{b^2(t_e)}(c^2 - V_e^2)} = \sqrt{c^2 - \frac{b^2(t)}{b^2(t_e)}(c^2 - \frac{a^2(t)v_e^2}{b^2(t)})} \tag{28}$$

$$\frac{a(t)}{b(t)}v_s(t) = V_s(t) = \sqrt{c^2 - \frac{b^2(t)}{b^2(t_s)}(c^2 - V_s^2)} = \sqrt{c^2 - \frac{b^2(t)}{b^2(t_s)}(c^2 - \frac{a^2(t)v_s^2}{b^2(t)})}. \tag{29}$$

Assuming the distance between light source and the earth to be L_0 at t_0, and to be L at t, then the following relation can be obtained

$$L_0 = L + \int_t^{t_0} v_s(t)dt - \int_t^{t_0} v_e(t)dt$$

$$= L + \int_t^{t_0} \frac{b(t)}{a(t)} \sqrt{c^2 - \frac{b^2(t)}{b^2(t_s)}(c^2 - \frac{a^2(t)v_s^2}{b^2(t)})} \cdot dt$$
$$- \int_t^{t_0} \frac{b(t)}{a(t)} \sqrt{c^2 - \frac{b^2(t)}{b^2(t_e)}(c^2 - \frac{a^2(t)v_e^2}{b^2(t)})} \cdot dt. \qquad (30)$$

This is just the kinematics law assumed to be equivalent to the geodesic motion of the gravity-free particle in the homogeneous expanding universe.

We assume that the light source emits a light signal at t, which is received by the observer in earth at t_0. According to the non-stationary cosmological metric, the propagation of the light signal satisfies the zero geodesic principle. We can directly write down the propagation distance D_H of the photon from t to t_0,

$$\int_t^{t_0} \frac{b(t)}{a(t)} \cdot cdt = \int_r^{r_0} \frac{dr}{\sqrt{1-kr^2}} := D_H. \qquad (31)$$

According to previous analysis of distance between the light source and the earth observer, we obtain

$$D_H = L - \int_t^{t_0} \frac{b(t)}{a(t)} \sqrt{c^2 - \frac{b^2(t)}{b^2(t_e)}(c^2 - \frac{a^2(t)v_e^2}{b^2(t)})} dt. \qquad (32)$$

The minus in above equation results from definition of positive direction for the coordinates of the universal inertial reference frame, here the speed of the earth is assumed to be toward the light source.

After the kinematics law of the commoving point related with observation theory is presented, we can discuss the *chain condition* between the geodesics of an instantaneous metric solution of t and the distribution of the gravitational matter in the next moment $t + dt$. First, in a matter-dominated and homogenous isotropic universe, according to the energy conservation principle, the energy of the matter contained in the range L_0 at t_0 should equal to that in the range L at t,

$$\rho_{t_0} \cdot L_0^3 = \rho_t \cdot L^3. \qquad (33)$$

Second, we can directly apply Eqs.(30)-(32) of the observation theory, and then

$$L = \int_t^{t_0} \frac{b(t)}{a(t)} \cdot cdt + \int_t^{t_0} \frac{b(t)}{a(t)} \sqrt{c^2 - \frac{b^2(t)}{b^2(t_e)}(c^2 - \frac{a^2(t)v_e^2}{b^2(t)})} \cdot dt; \qquad (34)$$
$$L_0 = \int_t^{t_0} \frac{b(t)}{a(t)} \cdot cdt + \int_t^{t_0} \frac{b(t)}{a(t)} \sqrt{c^2 - \frac{b^2(t)}{b^2(t_s)}(c^2 - \frac{a^2(t)v_s^2}{b^2(t)})} \cdot dt; \qquad (35)$$

After that, the connection between the instantaneous space-time geometry in the homogenous expanding universe and the matter evolution at the next moment satisfy the following condition

$$\rho_t = \rho_{t_0} \cdot \frac{L_0^3}{L^3} = \rho_{t_0} [\frac{\int_t^{t_0} \frac{b(t)}{a(t)} \cdot cdt + \int_t^{t_0} \frac{b(t)}{a(t)} \sqrt{c^2 - \frac{b^2(t)}{b^2(t_s)}(c^2 - \frac{a^2(t)v_s^2}{b^2(t)})} \cdot dt}{\int_t^{t_0} \frac{b(t)}{a(t)} \cdot cdt + \int_t^{t_0} \frac{b(t)}{a(t)} \sqrt{c^2 - \frac{b^2(t)}{b^2(t_e)}(c^2 - \frac{a^2(t)v_e^2}{b^2(t)})} \cdot dt}]^3. \qquad (36)$$

Therefore, Eqs.(20)-(21) and (36) actually constitute a complete equations group to solve the evolution law of the non-stationary universe. Here, $\{t_e, v_e\}$ and $\{t_s, v_s\}$ correspond to the initial speed of gravity-free points at the initial moment, which can be looked as the initial condition of the cosmology.

REVISION OF THE OBSERVATIONAL THEORY

We know that one of the theoretical bases of observational cosmology is Hubble theorem, the relationship between the metric geometry and the observed photon's red-shift. Without explicitly solving the fundamental equations Eqs.(20-21) for the dynamics of the expansion, it is still possible to understand the kinematic effects (see details in [6]) from the gravity-geometrized metric model (17). First, we assume the absolute luminosity of the light source is $\mathcal{L}$, and luminosity distance can be defined by its observed light flux. The absolute luminosity is defined as the energy

emitted from the light source per unit time of the reference frame rest on the light source. The light flux is defined as the energy received by the detector on the earth per area per unit time. According to the conservation of the light's sphere energy propagating in three dimensional space. We have

$$D_L^2 := \frac{\mathcal{L}}{4\pi\mathcal{F}} \tag{37}$$

Now we turn to the kinematical problem of light propagation. Because the light source and the earth can be treated as the gravity-free commoving point, propagation distance of the light signal received by the observe on earth at the moment after t_0 can be obtained by taking into consideration of the respective displacement of the light source and the earth observer relative to the background space-time.

$$\begin{aligned}\int_{t+\delta t}^{t_0+\delta t_0} c\frac{b(t)dt}{a(t)} &= D_H + [-v_e(t_0)\delta t_0 + v_s(t)\delta t] \\ &= D_H + [-\frac{b(t_0)}{a(t_0)}\sqrt{c^2 - \frac{b^2(t_0)}{b^2(t_e)}(c^2 - \frac{a^2(t_0)v_e^2}{b^2(t_0)})}\delta t_0 \\ &\quad + \frac{b(t)}{a(t)}\sqrt{c^2 - \frac{b^2(t)}{b^2(t_s)}(c^2 - \frac{a^2(t)v_s^2}{b^2(t)})}\delta t].\end{aligned} \tag{38}$$

Where $-v_e(t_0)\delta t_0$ is the displacement of the earth observer relative to background space-time within Δt_0 at t_0, similarly, $v_s(t)\delta t$ is the displacement of the light source relative to background space-time within Δt at t . Substituting Eq.(31) into Eq. (38), we have

$$\begin{aligned}\int_{t}^{t+\delta t} c\frac{b(t)dt}{a(t)} + \int_{t+\delta t}^{t_0} c\frac{b(t)dt}{a(t)} &= \int_{t+\delta t}^{t_0} c\frac{b(t)dt}{a(t)} + \int_{t_0}^{t_0+\delta t_0} c\frac{b(t)dt}{a(t)} \\ &\quad -[-v_e(t_0)\delta t_0 + v_s(t)\delta t].\end{aligned} \tag{39}$$

Applying the median theorem, we have

$$(c\frac{b(t)}{a(t)} + v_s(t))\delta t = (c\frac{b(t_0)}{a(t_0)} + v_e(t_0))\delta t_0. \tag{40}$$

In the case of non-stationary metric, the time interval of the same waveband at t will be not consistent with that at t_0, its connection is given by Eq.(40). Similarly, we take the event reading of the light signal emitted at t as 1, namely $b(t)dt = 1$, the lasting time is $dt = 1/b(t)$, according to previous obtained propagation law, the observer at t_0 can detect the light signal in the time interval of the $dt_0 = \frac{cb(t)/a(t)+v_s(t)}{cb(t_0)/a(t_0)+v_s(t_0)}dt$. Hence the event reading the same light signal at t_0 is $b(t_0)dt_0 = \frac{c/a(t)+v_s(t)/b(t)}{c/a(t_0)+v_s(t_0)/b(t_0)}$. Considering the local event speed reading of the light source at t is $V_s(t)$, and the local event speed reading of earth observer at t_0 is $V_e(t_0)$. Therefore, the Lorenz transform which is caused by these local event speed is $\gamma = \frac{1}{\sqrt{1-[V_s(t)-V_e(t_0)]^2}}$. The ultimate frequency shift of a light signal emitted from the light source and received by the earth observer is,

$$1 + z = \frac{a(t_0)b(t_0)}{a(t)b(t)} \cdot \gamma \cdot (\frac{cb(t) + v_s(t)a(t)}{cb(t_0) + v_e(t_0)a(t_0)}). \tag{41}$$

On the other hand, the absolute luminosity $\mathcal{L}$ appeared in the luminosity distance expression ${d_L}^2 = \frac{\mathcal{L}}{4\pi\mathcal{F}}$ of astronomical observation theory should be described by unit event reading of time $b(t)dt$ (first kind of explanation). Here we assume the effect of the gravity is really originated from the change of the space-time interval of proper events at the points of different matter density, thus we must apply a global coordinate transformation for introducing the physical quantity of event reading of time. We can define

$$-b^2(t)dt^2 \longrightarrow -dT^2, \tag{42}$$

Because here we consider only the kinetic law, the above mathematic transform wouldn't change the nature of kinematics(Strictly speaking, in the processing of the astronomical data from the supernova, what should be put into analyzing together are those detected *on the same time* by the observer rest on the Earth because that the redefinition

of (42) is available only for once in the same processing). After that, the metric (17) immediately recovers to following form

$$ds^2 = -dT^2 + a^2(T)[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2]. \tag{43}$$

Thus we can apply the mathematic techniques of conventional observation theory. According to Eq.(42), after we make corresponding coordinates transform for the frequency shift in Eq.(41) we can obtain $1+z = \frac{a(T_0)}{a(T)} \cdot \gamma \cdot (\frac{c+v_s(T)a(T)}{c+v_e(T_0)a(T_0)})$.

In addition, considering the physical requirement included in the definitions of light flux and absolute luminosity, we have

$$D_L^2 = D_H^2(1+z)^2. \tag{44}$$

That is to say, redshift equal to the effect that the light energy passing through two different sphere of light source is reduced to $(1+z)^2$ times per unit time, where one factor $(1+z)$ comes from the decrease of the photon energy caused by red-shift, the another factor $(1+z)$ comes from the decrease of the photon number detected per unit time caused by the increase of the corresponding interval background time for a photon propagating from the light source to the earth.

According to the mathematical method of traditional observation theory, if we define $1+z' = \frac{a(T_0)}{a(T)} = (\frac{c+v_e(T_0)a(T_0)}{c+v_s(T)a(T)}) \cdot (\frac{1+z}{\gamma})$, we can easily obtain

$$\begin{aligned} D_H = \int_T^{T_0} \frac{dT}{a(T)} &= a^{-1}(T_0)H_0^{-1}[z' - \frac{1}{2}(1+q_0)z'^2 + (\frac{1}{3} + \frac{5}{3}q_0 - \frac{j_0}{6} + \frac{q_0^2}{2})z'^3 \\ &+(-1 - \frac{21q_0}{4} + \frac{5}{6}j_0 + \frac{s_0}{24} + \frac{5}{12}q_0 j_0 - \frac{15}{4}q_0^2 - \frac{5}{8}q_0^3 + 1 + 2q_0 \\ &-\frac{j_0}{6} + \frac{q_0^2}{2} - 3(1+\frac{q_0}{2})(\frac{q_0}{6} + \frac{1}{3}))z'^4 + ...] \end{aligned} \tag{45}$$

Here cosmological parameters: Hubble factor H, decelerating factor q et al, satisfy

$$H := \frac{\dot{a}(T)}{a(T)}; \tag{46}$$

$$q := -\frac{\ddot{a}(T)}{a(T)H^2} \tag{47}$$

$$j := \frac{a^{(3)}(T)}{a(T)\cdot H^3} \tag{48}$$

$$s := \frac{a^{(4)}(T)}{a(T)\cdot H^4} \tag{49}$$

Substituting Eq. (45) into Eq. (44), we obtain

$$H_0 D_L = a^{-1}(T_0)\frac{1+z}{1+z'} \cdot [z' + \frac{1}{2}(1-q_0)z'^2 + \frac{1}{6}(q_0 + 3q_0^3 - 1 - j_0)z'^3 + ...]. \tag{50}$$

Because the practical observation is compared with the earth observer, if cosmological metric (17) of universal inertial observer is scaled by the standard rule of the earth, it automatically require that $a(T_0) = 1$, after substitution of the relation between z' and the practical red-shift z, we have

$$\begin{aligned} H_0 D_L &= a^{-1}(T_0) \cdot \gamma \cdot (\frac{c+v_s(T)a(T)}{c+v_e(T_0)a(T_0)}) \cdot \{[\frac{(c+v_e(T_0)a(T_0))}{(c+v_s(T)a(T))} \cdot (\frac{1+z}{\gamma}) - 1] \\ &+\frac{1}{2}(1-q_0)[\frac{(c+v_e(T_0)a(T_0))}{(c+v_s(T)a(T))} \cdot (\frac{1+z}{\gamma}) - 1]^2 \\ &+\frac{1}{6}(q_0 + 3q_0^3 - 1 - j_0)[\frac{(c+v_e(T_0)a(T_0))}{(c+v_s(T)a(T))} \cdot (\frac{1+z}{\gamma}) - 1]^3 + ...\}. \end{aligned} \tag{51}$$

Here, the relation between the red-shift of specific galaxy and its initial speed has been given by Eq.(41). The distribution of initial speed v of commoving points is the initial condition of homogenous cosmology.

On the other hand, in our theory, what the Hubble's law carried out from the supernova data and CMB [3, 4, 5] is only the law of $\frac{d^2a(T)}{dT^2}$ rather than $\frac{d^2a(t)}{dt^2}$. It should be noted that the Hubble parameter in Eq.(51) doesn't denote the speed of commoving distance change. The practical change of the relative distance between commoving points satisfies Eq.(30). In addition, the cosmological parameters in Eq.(51) are defined by the event reading of time, we must use the representation of the background coordinate time of universal initial observer to solve the geometrized equations (20)-(21) of cosmic gravity. According to Eq.(42), the conversion relation between the background coordinates reading and the event reading of the function of cosmological scale factor satisfies,

$$\frac{da(T)}{dT} = \frac{a(T_0)-a(T)}{T_0-T} = \frac{da(t)}{dt}\cdot\frac{dt}{dT} = \frac{da(t)}{dt}\cdot b^{-1}(t); \tag{52}$$

$$\begin{aligned}\frac{d^2a(T)}{dT^2} &= \frac{d^2a(t)}{dt^2}\cdot(\frac{dt}{dT})^2 + \frac{da(t)}{dt}\cdot\frac{d^2t}{dT^2}\\ &= \frac{d^2a(t)}{dt^2}\cdot b^{-2}(t) + \frac{da(t)}{dt}\cdot(-b^{-3}(t)\frac{db(t)}{dt}).\end{aligned} \tag{53}$$

REMARKS

In summary, we have discussed the geometrical equivalence of the cosmological gravity resorting to the method of inertial reference frame, which is carried out by the principle of consistent relativity for constructing the dynamics equation [1]. And we have introduced two independent geometrical quantities according to cosmological principle and the hypothesis of perfect fluid. On this basis, the fundamental equation for the geometry of cosmology was given. While the universe is a non-stationary matter field, in which the commoving points are in gravity free. Therefore, instantaneous distribution of matter field is determined by instantaneous space-time geometry, at the same time, instantaneous space-time geometry would give the matter distribution at the next moment according to geodesic motion of the gravity-free system. After that, we discussed the motion of the gravity-free points in cosmology by use of the proper energy conservation law in similar to Special relativity. The advantage of this method is that the amendment of mass caused by the change of velocity is taken into consideration, is different to the traditional mechanical energy conservation principle. Furthermore, the chain condition of the matter field at the next moment and the metric geometry are given mathematically. At last, we also discussed the function relation between metric geometry and red-shift, which are the basis of observation theory of cosmology. While the geometry quantity in the metric derived in this paper no longer denotes the commoving distance, and the practical commoving distance is not only related with metric geometry, but also related with the initial condition of the motion of specific galaxy.

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